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Non-Abelian T-Duality from Penrose Limit of the Pilch-Warner Solution

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Abstract. In this paper we consider the non-abelian T-duality (NATD) of a certain plane wave background, namely the Penrose limit of the Pilch-Warner solution. The newly obtained T-dual pp-wave metric and NS two-form are naturally more complicated from those in the original theory, because NATD procedure mixes the coordinates in a non-trivial way. We also derive the NAT-dual R-R sector by applying the Fourier-Mukai transform on the corresponding pp-wave R-R fluxes.

1 Introduction

Dualities play an important role in modern physics and especially in string theory. In general, the term duality refers to a situation where two seemingly different physical theories turn out to be equivalent in a nontrivial way.

In the mid-1980's it was noticed that a string propagating on a circle of radius R is physically equivalent to a string propagating on a circle of radius $1/R$. This phenomenon is now known as T-duality. In essence, T-duality provides different mathematical descriptions of the same physical system. In other words, all observable quantities in one description are identified with quantities in the dual description. In this context T-duality is also a perturbative duality relating the weak coupling regimes of both theories. Therefore one can test it in perturbation theory via comparison of the corresponding string spectra.

There are different approaches to abelian T-duality mainly due to the work of Buscher [1, 2]. In general, for any two-dimensional nonlinear sigma model with certain isometry group (abelian or not) there exists a clear procedure for obtaining the T-duality transformation rules. Firstly, one has to gauge the group structure by introducing Lagrange multipliers and auxiliary gauge fields into the Lagrangian of the theory. The equations of motion for the multipliers force the field strength to vanish. Secondly, substituted in the gauged Lagrangian, the solutions of those equations reproduce the original theory. Finally, one can integrate out the auxiliary gauge fields and interpret the Lagrange multipliers as dual coordinates thus arriving at the (non-)abelian T-dual theory. Unlike the abelian case, the non-abelian T-duality (NATD) has very distinctive properties. One remarkable feature is that applying NATD on a geometry, which has some explicit isometry, one typically obtains dual solution in which this isometry is no longer present. Therefore this transformation procedure is not easily invertible. Furthermore performing second T-duality could lead to completely different theory than the original one. Thus one can infer that NATD is not a symmetry of a conformal field theory, but a symmetry between different conformal field theories.

Together with the metric and the NS two-form, the R-R fields also transform under non-abelian T-duality. Although various methods have been developed for performing this transformation, in this paper we adopt the approach where T-duality can be lifted to K-theory as a Fourier-Mukai transform with Poincare bundle as a kernel. It has been shown [3] that Nahm transform of instantons is related to the transformation of D-branes under T-duality and the concrete isomorphism between K-theory groups, which realises this transformation on a torus of arbitrary dimensions, has been identified. On the other hand, the equations of motion for a CFT with topological defects imply a direct connection between the Poincare line bundle and the boundary conditions affected by T-duality [4]. Since boundary conditions correspond to D-branes (which are sources of R-R charges), one can define an action of a topological defect on the R-R charges. This action is of Fourier-Mukai type with kernel given by the gauge invariant flux of the defect [5].

A strong motivation for performing this work comes from the achievements of the AdS/CFT correspondence [6]. This is a framework providing a successful non-perturbative approach to strongly coupled Yang-Mills theories via the properties of their dual classical supergravity solutions and vice versa. A well-known example of strongly coupled gauge theory is quantum chromodynamics, which at low energies does not allow any perturbative treatments. Therefore one is forced to look for an alternative non-perturbative description of the theory in the context of the gauge/gravity duality. The problem of producing more realistic QCD-like theories can be reduced to finding highly nontrivial background solutions that break significantly the amount of supersymmetry and conformal symmetry. An example of such non-trivial supergravity background is the Pilch-Warner (PW) geometry [7], which is holographically dual to $\mathcal{N} = 1$ Leigh-Strassler theory [8].

Second motivation for this work comes from the fact that string theory in pp-waves backgrounds simplifies due to the existence of a natural light-cone gauge, and in many cases can be exactly solved and quantized [9]. Furthermore these solutions model radiation moving at the speed of light and may consist of electromagnetic or gravitational radiation, which is important in the light of the recent discovery of gravitational waves [10, 11]. Finally, the gauge interactions we detect in nature are characterized by a scale and hence are not conformally invariant. Thus, an essential step to take is to apply the Penrose-Güven limit [12,13] on SUGRA backgrounds associated with non-conformal gauge theories. These and other considerations suggest that obtaining and analysing new pp-wave limits of complicated backgrounds can provide insights in many aspects of the theory.

The Penrose limit (PL) is defined as a specific scaling of the metric and supergravity fields along a null geodesic [12, 13]. This limit encodes diffeomorphism invariant information, such as the rate of growth of the curvature and the geodesic deviation along a null geodesic of the original space-time. Recently the authors of [14] obtained two different Penrose limits of the Pilch-Warner NAT-dual background solution. This solutions are much simpler than the one we found in this paper. In [14] the procedure is as follows. First, one considers the NATD of the type IIB PW solution and ends up with a highly nontrivial type IIA background. Then, one finds an appropriate Penrose limit of the new NAT-dual solution. In this paper we proceed by interchanging the order of the procedures. First we find a Penrose limit of the PW solution, which retains the $SU(2)$ group structure of the original metric. Then we apply the NATD procedure on the G-structure of this pp-wave solution.

This paper is organised as follows. In Section 2 we give a brief review of the general setup of non-abelian T-duality following the method of gauging the nonabelian background isometries. Then, in Section 3 we consider the special case of Pilch-Warner supergravity solution. The background geometry consists of warped AdS_5 space times squashed five-sphere. In Section 4 we find a pp-wave limit of the PW geometry around $\theta = \pi/2$ null geodesics. One observes that the pp-wave metric has manifest $SU(2)$ symmetry. This allows us to apply the NATD procedure to this particular plane wave geometry. Here we consider the NS sector in Section 5, and subsequently the R-R sector in section 6. Finally, in section 7 we conclude with a short summary of our results.

2 Transformation rules for NATD

2.1 The NS sector

Our consideration begins with a brief review of the general setup of non-abelian T-duality [5] (see also [15–17]). The general transformation rules under NATD for a given supergravity solution with non-trivial NS two-form B follows from the group structure G with generators T^a and structure constants f^a_{bc}, $a = 1, \ldots, \dim(G)$, which is supported by the background. Let θ^k, are the coordinates describing the G-part of the metric. Then we can write the metric and the B-field in the following way:

$$\mathrm{d}s^2 = G_{\mu\nu}(Y)\mathrm{d}Y^\mu \mathrm{d}Y^\nu + 2G_{\mu a}\Omega^a_k \mathrm{d}Y^\mu \mathrm{d}\theta^k + G_{ab}\Omega^a_m \Omega^b_k \mathrm{d}\theta^m \mathrm{d}\theta^k, \tag{1}$$

$$B = \frac{1}{2}B_{\mu\nu}(Y)\mathrm{d}Y^\mu \mathrm{d}Y^\nu + B_{\mu a}\Omega^a_k \mathrm{d}Y^\mu \mathrm{d}\theta^k + \frac{1}{2}B_{ab}\Omega^a_m \Omega^b_k \mathrm{d}\theta^m \mathrm{d}\theta^k. \tag{2}$$

For any group element $g \in G$ one can construct the Maurer-Cartan one-form

$$g^{-1}\mathrm{d}g = L^a T_a = \Omega^a_k \mathrm{d}\theta^k T_a, \tag{3}$$

where the left-invariant one-forms L^a satisfy the equation

$$\mathrm{d}L^a = -\frac{1}{2}f^a_{bc}L^b L^c, \quad \text{or} \tag{4}$$

$$\partial_i \Omega^c_j - \partial_j \Omega^c_i = -f^c_{ab}\Omega^a_i \Omega^b_j. \tag{5}$$

It is convenient to use the following form of the Lagrangian:

$$\mathcal{L} = Q_{\mu\nu}\partial Y^\mu \bar\partial Y^\nu + Q_{\mu a}\Omega^a_k \partial Y^\mu \bar\partial\theta^k + Q_{a\mu}\Omega^a_k \partial\theta^k \bar\partial Y^\mu + Q_{ab}\Omega^a_m \Omega^b_k \partial\theta^m \bar\partial\theta^k, \tag{6}$$

where the Q's are given by

$$Q_{\mu\nu} = G_{\mu\nu} + B_{\mu\nu}, \quad Q_{\mu a} = G_{\mu a} + B_{\mu a}, \quad Q_{ab} = G_{ab} + B_{ab}. \tag{7}$$

The next step is to gauge the group structure by introducing Lagrange multipliers x^a and gauge fields A^a in the Lagrangian of the original theory

$$\mathcal{L} = Q_{\mu\nu}\,\partial Y^\mu \bar\partial Y^\nu + Q_{\mu a}\,\partial Y^\mu \bar A^a + Q_{a\mu}A^a\,\bar\partial Y^\mu + Q_{ab}A^a \bar A^b - x^a(\partial \bar A^a - \bar\partial A^a + f^a_{bc}A^b \bar A^c). \tag{8}$$

With this gauged Lagrangian one can easily reproduce the original theory by simply eliminating the Lagrangian multipliers x^a. The equations of motion $\delta\mathcal{L}/\delta x^a = 0$ directly imply vanishing field strength

$$F^a_{+-} = \partial \bar A^a - \bar\partial A^a + f^a_{bc}A^b \bar A^c = 0, \tag{9}$$

which has the obvious solutions

$$A^a = \Omega^a_k\,\partial\theta^k, \quad \bar A^a = \Omega^a_k\,\bar\partial\theta^k. \tag{10}$$

Plug in back these solutions into (8) we get back to the original theory (6).

On the other hand, to obtain the T-dual theory, we have to integrate out the gauge fields A^a. This leads to the following conditions (up to boundary terms):

$$Q_{\mu a}\,\partial Y^\mu + Q_{ba}A^b - x^c f^c_{ba}A^b + \partial x^a = 0, \tag{11}$$
$$Q_{a\mu}\,\bar\partial Y^\mu + Q_{ab}\bar A^b - x^c f^c_{ab}\bar A^b - \bar\partial x^a = 0. \tag{12}$$

The solutions of these conditions are

$$A^a = -M^{-1}_{ba}(Q_{\mu b}\,\partial Y^\mu + \partial x^b), \tag{13}$$
$$\bar A^a = M^{-1}_{ab}(\bar\partial x^b - Q_{b\mu}\,\bar\partial Y^\mu), \tag{14}$$

where we have defined the structure matrix

$$M_{ab} = Q_{ab} - x^c f^c_{ab}. \tag{15}$$

To obtain the NATD theory we plug in A^a and $\bar A^a$ back into (8). The resulting Lagrangian is

$$\hat{\mathcal{L}} = \widehat{E}_{\mu\nu}\,\partial Y^\mu\bar\partial Y^\nu + \widehat{E}_{\mu a}\,\partial Y^\mu\bar\partial x^a + \widehat{E}_{a\mu}\,\partial x^a\bar\partial Y^\mu + \widehat{E}_{ab}\,\partial x^a\bar\partial x^b, \tag{16}$$

where we have introduced the following notations:

$$\begin{aligned} \widehat{E}_{\mu\nu} &= Q_{\mu\nu} - Q_{\mu a}M^{-1}_{ab}Q_{b\nu}, & \widehat{E}_{\mu a} &= Q_{\mu b}M^{-1}_{ba}, \\ \widehat{E}_{a\mu} &= -Q_{b\mu}M^{-1}_{ab}, & \widehat{E}_{ab} &= M^{-1}_{ab}. \end{aligned} \tag{17}$$

The quantities of the two pictures are related by (10) and (13), (14). The comparison gives

$$\Omega^a_k\,\partial\theta^k = -M^{-1}_{ba}(Q_{\mu b}\,\partial Y^\mu + \partial x^b), \tag{18}$$
$$\Omega^a_k\,\bar\partial\theta^k = M^{-1}_{ab}(\bar\partial x^b - Q_{b\mu}\,\bar\partial Y^\mu). \tag{19}$$

Finally, the dual field content in the NS sector can be extracted by separating symmetric and antisymmetric parts of the quantity $\widehat{E}_{AB}$. The result is

$$\widehat{G}_{\mu\nu} = G_{\mu\nu} - \frac{1}{2}\left(Q_{\mu a}M^{-1}_{ab}Q_{b\nu} + Q_{\nu a}M^{-1}_{ab}Q_{b\mu}\right), \tag{20}$$
$$\widehat{G}_{\mu a} = \frac{1}{2}\left(Q_{\mu b}M^{-1}_{ba} - Q_{b\mu}M^{-1}_{ab}\right), \tag{21}$$
$$\widehat{G}_{ab} = \frac{1}{2}\left(M^{-1}_{ab} + M^{-1}_{ba}\right), \tag{22}$$
$$\widehat{B}_{\mu\nu} = B_{\mu\nu} - \frac{1}{2}\left(Q_{\mu a}M^{-1}_{ab}Q_{b\nu} - Q_{\nu a}M^{-1}_{ab}Q_{b\mu}\right), \tag{23}$$
$$\widehat{B}_{\mu a} = \frac{1}{2}\left(Q_{\mu b}M^{-1}_{ba} + Q_{b\mu}M^{-1}_{ab}\right), \tag{24}$$
$$\widehat{B}_{ab} = \frac{1}{2}\left(M^{-1}_{ab} - M^{-1}_{ba}\right). \tag{25}$$

The above expressions define a NATD procedure which gives the dual of geometry possessing some non-abelian symmetry group G acting without isotropy. The group structure is implicitly encoded in the matrix M_{ab} through the structure constants f_{ab}^c. The fact that the group acts without isotropy is crucial because this allows us to completely fix the gauge by algebraic conditions on the target space coordinates. Consequently, we are able to clearly distinguish between dual and original coordinates. In Section 5 we will use this procedure to obtain the NATD for the particular case of $SU(2)$ group structure supported by the pp-wave limit of the Pilch-Warner geometry.

2.2 The RR sector

The transformation rules of the R-R field strengths under NATD have been worked out recently using Fourier-Mukai transform for the case of backgrounds possessing $SU(2)$ symmetry acting without isotropy. Our aim is to use this procedure for the pp-wave limit of the PW geometry, which has the same symmetry. The concrete formula obtained in [5] states

$$\widehat{\mathcal{G}} = \int_G \mathcal{G} \wedge e^{\widehat{B} - B - dx^a \wedge L^a + \frac{1}{2} x^a f^a_{bc} L^b \wedge L^c}, \tag{26}$$

where $\mathcal{G} = \sum_p \mathcal{G}_p$ is the sum of the gauge invariant RR p-form field strengths and $\widehat{\mathcal{G}}$ is the sum of the dual ones; the index p takes even values for type IIA and odd values for type IIB supergravities. We will work in the co-frame of left-invariant one-forms L^a, in which $\mathcal{G}$ can be always represented as sum of differential forms that do not contain any L^a, differential forms that contain one L^a, differential forms that contain wedge product of two L^a's, and differential forms that contain wedge products of three L^a's

$$\mathcal{G} = \mathcal{G}^{(0)} + \mathcal{G}^{(1)}_a \wedge L^a + \frac{1}{2} \mathcal{G}^{(2)}_{ab} \wedge L^a \wedge L^b + \mathcal{G}^{(3)} \wedge L^1 \wedge L^2 \wedge L^3. \tag{27}$$

We will apply this in Section 6 to find the NAT-dual RR fields for the pp-wave metric of the PW solution.

3 The Pilch-Warner solution

The type IIB Pilch-Warner (PW) background [7] is a solution to the $\mathcal{N} = 8$ five-dimensional gauged supergravity lifted to ten dimensions. In the ultra–violet (UV) critical point of the flow, it gives the standard maximally supersymmetric AdS_5 geometry, while in the IR it provides a gravity dual of $\mathcal{N} = 4$ SYM theory, which is softly broken down to $\mathcal{N} = 2$ supersymmetry [18, 19]. In this paper we adopt the setup given in [20], where the original solution is shifted in one of its global $U(1)_\phi$ isometries in a way that cancels the factor of $e^{i\,\phi}$ in the A_2 potential[1]. The metric of the 10-dimensional PW solution is given by [20]

$$\mathrm{d}s^2 = \Omega^2\, \mathrm{d}s^2_{AdS_5} + \mathrm{d}s^2_{SQ^5}\,. \tag{28}$$

Here one has the warped AdS part

$$\mathrm{d}s^2_{AdS_5} = L^2\, \left(-\cosh^2\!\rho\, \mathrm{d}\tau^2 + \mathrm{d}\rho^2 + \sinh^2\!\rho\, \mathrm{d}\Omega_3^2\right)\,, \tag{29}$$

where $\mathrm{d}\Omega_3^2 = \mathrm{d}\phi_1^2 + \sin^2\!\phi_1\, \left(\mathrm{d}\phi_2^2 + \sin^2\!\phi_2\, \mathrm{d}\phi_3^2\right)$ is the metric on the unit 3-sphere, and the squashed 5-sphere

$$\begin{aligned}\mathrm{d}s^2_{SQ^5} = \frac{2}{3}\, L^2\, \Omega^2\, \Bigg[&\mathrm{d}\theta^2 + \frac{4\cos^2\!\theta}{3-\cos 2\theta}\left(\sigma_1^2+\sigma_2^2\right) + \frac{4\sin^2\!2\theta}{(3-\cos 2\theta)^2}\left(\sigma_3+\mathrm{d}\phi\right)^2\\ &+\frac{2}{3}\left(\frac{1-3\,\cos 2\theta}{\cos 2\theta - 3}\right)^2\left(\mathrm{d}\phi - \frac{4\cos^2\!\theta}{1-3\,\cos 2\theta}\,\sigma_3\right)^2\Bigg]\,.\end{aligned} \tag{30}$$

In the above expressions one also has the warp factor,

$$\Omega^2 = 2^{1/3}\left(1-\frac{1}{3}\,\cos 2\theta\right)^{1/2}, \tag{31}$$

and the left-invariant $SU(2)$ one-forms,

$$\begin{aligned}\sigma_1 &= \frac{1}{2}\left(\sin\beta\, \mathrm{d}\alpha - \cos\beta\, \sin\alpha\, \mathrm{d}\gamma\right),\\ \sigma_2 &= -\frac{1}{2}\left(\cos\beta\, \mathrm{d}\alpha + \sin\beta\, \sin\alpha\, \mathrm{d}\gamma\right),\\ \sigma_3 &= \frac{1}{2}\left(\mathrm{d}\beta + \cos\alpha\, \mathrm{d}\gamma\right),\end{aligned} \tag{32}$$

which satisfy $\mathrm{d}\sigma_i = \varepsilon_{ijk}\,\sigma_j\wedge\sigma_k$, so the metric on the unit 3-sphere is $\mathrm{d}s^2_{S^3} = \sigma_i\,\sigma_i$. The PW background also contains non-trivial NS-NS and R-R fluxes, which satisfy the following equations of motion and Bianchi identities [21]:

$$\begin{gathered}\Phi = C_0 = 0, \quad F_1 = \mathrm{d}C_0 = 0\,,\\ C_2 = \mathrm{Re}(\mathrm{A}_2), \quad \mathrm{B}_2 = \mathrm{Im}(\mathrm{A}_2)\,,\\ H_3 = \mathrm{d}B_2, \quad F_3 = \mathrm{d}C_2 - C_0\wedge H_3 = \mathrm{d}C_2\,,\\ \mathrm{d}F_3 = \mathrm{d}H_3 = 0, \quad \mathrm{d}F_5 = H_3\wedge F_3\,,\\ \mathrm{d}(\star F_3) = -H_3\wedge F_5, \quad \mathrm{d}(\star H_3) = F_3\wedge F_5, \quad F_5 = \star F_5\,,\end{gathered}$$

[1] This fact will considerably simplify the later analysis of the T-dual geometry.

$$
\begin{aligned}
\mathrm{d}C_4 + \mathrm{d}\tilde{C}_4 &= F_5 + C_2 \wedge H_3\,, \\
F_7 = \star F_3 &= \mathrm{d}C_6 - C_4 \wedge H_3\,, \\
F_9 = \star F_1 = 0 = \mathrm{d}C_8 - C_6 \wedge H_3 &= C_6 \wedge H_3, \quad \chi = C_8 = 0\,.
\end{aligned} \tag{33}
$$

Here the field strengths are defined in terms of the corresponding potentials as

$$
H_3 \equiv \mathrm{d}B_2, \quad F_p \equiv \mathrm{d}C_{p-1} - C_{p-3} \wedge H_3. \tag{34}
$$

The only trivial is the dilaton (Φ) /axion (C_0) system of scalars. The first non-trivial fields are the NS-NS B_2-form field and the R-R 2-form potential C_2

$$
B_2 = -\frac{4}{9}\, 2^{1/3}\, L^2\, \cos\theta \left(\mathrm{d}\theta \wedge \sigma_1 + \frac{2\,\sin 2\theta}{3-\cos 2\theta}\, (\sigma_3 + \mathrm{d}\phi) \wedge \sigma_2 \right)\,, \tag{35}
$$

$$
C_2 = \frac{4}{9}\, 2^{1/3}\, L^2\, \cos\theta \left(\mathrm{d}\theta \wedge \sigma_2 - \frac{2\,\sin 2\theta}{3-\cos 2\theta}\, (\sigma_3 + \mathrm{d}\phi) \wedge \sigma_1 \right)\,. \tag{36}
$$

These fields can be encoded in the following 2-form complex potential:

$$
A_2 = C_2 + iB_2 = -\frac{4i 2^{1/3} L^2 \cos\theta}{9} \Big(\mathrm{d}\theta - \frac{2i \sin 2\theta}{3-\cos 2\theta} (\sigma_3 + \mathrm{d}\phi) \Big) \wedge (\sigma_1 + i\sigma_2)\,. \tag{37}
$$

One immediately notes that there is no factor of $e^{i\,\phi}$ in the above formula as opposed to the original solution in [7]. The solution also includes the self-dual 5-form flux, $F_5 = \star F_5$

$$
F_5 = -\frac{2^{5/3}}{3}\, L^4\, \cosh\rho \sinh^3\!\rho\, (1+\star)\, \mathrm{d}\tau \wedge \mathrm{d}\rho \wedge \epsilon(S^3_\phi)\,, \tag{38}
$$

where $\epsilon(S^3_\phi) = \sin^2\phi_1\, \sin\phi_2\, \mathrm{d}\phi_1 \wedge \mathrm{d}\phi_2 \wedge \mathrm{d}\phi_3$, and $\star$ is the Hodge star operator. Here we will explicitly give the expressions for the F_3, F_5 and F_7 fluxes, which will be used to calculate their corresponding Penrose limit. The F_3 field strength is given by

$$
F_3 = F_3^{(1)}\, \mathrm{d}\theta \wedge \sigma_1 \wedge \sigma_3 + F_3^{(2)}\, \mathrm{d}\phi \wedge \sigma_2 \wedge \sigma_3 + F_3^{(3)}\, \mathrm{d}\theta \wedge \mathrm{d}\phi \wedge \sigma_1\,, \tag{39}
$$

where

$$
\begin{aligned}
F_3^{(1)} &= \frac{64 \times 2^{1/3} L^2 \cos^3\theta}{9(3-\cos 2\theta)^2}, \\
F_3^{(2)} &= \frac{32 \times 2^{1/3} L^2 \cos^2\theta \sin\theta}{9(3-\cos 2\theta)}\,, \\
F_3^{(3)} &= \frac{F_3^{(1)}}{16\cos^2\theta} (11 - 20\cos 2\theta + \cos 4\theta)\,.
\end{aligned} \tag{40}
$$

The explicit form of the self-dual F_5 form flux is

$$F_5 = F_5^{(1)}\, \mathrm{d}\tau \wedge \mathrm{d}\rho \wedge \mathrm{d}\phi_1 \wedge \mathrm{d}\phi_2 \wedge \mathrm{d}\phi_3 + F_5^{(2)}\, \mathrm{d}\theta \wedge \mathrm{d}\phi \wedge \sigma_1 \wedge \sigma_2 \wedge \sigma_3 \,, \quad (41)$$

with the following components:

$$\begin{aligned} F_5^{(1)} &= -\frac{8 \times 2^{2/3}\, L^4}{3}\, \cosh\rho \sinh^3\!\rho \sin^2\!\phi_1\, \sin\phi_2 \,, \\ F_5^{(2)} &= -\frac{1024 \times 2^{2/3}\, L^4 \cos^3\!\theta\, \sin\theta}{81\,(3-\cos 2\theta)^2} \,. \end{aligned} \quad (42)$$

And finally, the $F_7 = \star F_3$ field strength is written by

$$F_7 = \mathrm{d}\tau\wedge\mathrm{d}\rho\wedge\mathrm{d}\phi_1\wedge\mathrm{d}\phi_2\wedge\mathrm{d}\phi_3\wedge\big(F_7^{(1)}\sigma_2 \wedge \sigma_3 + F_7^{(2)}\mathrm{d}\theta \wedge \sigma_1 + F_7^{(3)}\mathrm{d}\phi \wedge \sigma_2\big) \,, \quad (43)$$

where one has

$$\begin{aligned} F_7^{(1)} &= \frac{32\, L^6 \cos^2\!\theta\, \sin\theta\,(11-\cos 2\theta)}{27\,(3-\cos 2\theta)}\, \cosh\rho \sinh^3\!\rho \sin^2\!\phi_1\, \sin\phi_2 \,, \\ F_7^{(2)} &= \frac{4}{9}\, L^6\,(\cos 3\theta - 5\, \cos\theta)\, \cosh\rho \sinh^3\!\rho \sin^2\!\phi_1\, \sin\phi_2 \,, \\ F_7^{(3)} &= -\frac{256\, L^6 \cos^2\!\theta\, \sin\theta}{27\,(3-\cos 2\theta)}\, \cosh\rho \sinh^3\!\rho \sin^2\!\phi_1\, \sin\phi_2 \,. \end{aligned} \quad (44)$$

All the non-trivial NS and R-R fluxes remain non-zero in the corresponding Penrose limit[2].

4 The Penrose limit of the Pilch-Warner solution

Following ref. [22], we are going to find a new Penrose limit of the shifted solution[3] (29). The geodesics lie near $\rho = 0,\ \theta = \pi/2$, where one has $x^- \sim L^2\ (\tau - 2\,\phi/3)$. We can Introduce the following coordinate redefinitions:

$$\begin{aligned} \rho &= \frac{3^{1/4}}{2^{2/3}}\, \frac{r}{L}, \quad \theta = \frac{\pi}{2} - \frac{3^{3/4}}{2 \times 2^{1/6}}\, \frac{y}{L}, \\ \tau &= \frac{3^{1/4}}{2 \times 2^{1/6}}\, \left(u + \frac{v}{L^2}\right), \quad \phi = \frac{3 \times 3^{1/4}}{4 \times 2^{1/6}}\, \left(u - \frac{v}{L^2}\right) \,, \end{aligned} \quad (45)$$

[2]The Penrose limit of the solution (29) is, in fact, one point in a family of pp-wave solutions. One can make more general Ansatze for the fluxes by introducing arbitrary constant parameters, and one can even introduce a non-constant dilaton and axion. The result is a large, multi-parameter space of solutions.

[3]There are several other papers, namely [20, 23], dealing with the Penrose limit of the PW solution, but they use different geodesics.

where $u = x^+$ and $v = x^-$ are the light-cone coordinates. In the limit $L \to \infty$ one finds the following metric[4]:

$$\mathrm{d}\tilde{s}^2 = -2\mathrm{d}u\mathrm{d}v - \frac{a}{4}(r^2+y^2)\mathrm{d}u^2 + \mathrm{d}y^2 + y^2\Big(\sigma_1^2+\sigma_2^2+\Big(\sigma_3+\frac{\sqrt{a}}{4}\,\mathrm{d}u\Big)^2\Big) \\ + \mathrm{d}r^2 + r^2\left(\mathrm{d}\phi_1^2+\sin^2\phi_1\left(\mathrm{d}\phi_2^2+\sin^2\phi_2\mathrm{d}\phi_3^2\right)\right)\,, \quad (46)$$

where $a = \sqrt{3}/2^{1/3}$. The Penrose limit of the 5-form (38) gives

$$\tilde{F}_5 = \tilde{F}_5^{(1)}\,\mathrm{d}u\wedge\mathrm{d}r\wedge\mathrm{d}\phi_1\wedge\mathrm{d}\phi_2\wedge\mathrm{d}\phi_3 + \tilde{F}_5^{(2)}\,\mathrm{d}u\wedge\mathrm{d}y\wedge\sigma_1\wedge\sigma_2\wedge\sigma_3\,, \quad (47)$$

where

$$\tilde{F}_5^{(1)} = -\frac{\sqrt{a}}{4}\,r^3\sin^2\phi_1\,\sin\phi_2,\quad \tilde{F}_5^{(2)} = -\frac{\sqrt{a}}{4}\,y^3\,\sin\phi_2\,. \quad (48)$$

The limit of the 2-form potential A_2 is given by

$$\tilde{A}_2 = \frac{y^2}{\sqrt{3}}\,\sigma_1\wedge\sigma_3 - \frac{3^{3/4}\,y^2}{4\times 2^{1/6}}\,\mathrm{d}u\wedge\sigma_1 - \frac{y}{\sqrt{3}}\,\mathrm{d}y\wedge\sigma_2 \\ + i\left(\frac{y^2}{\sqrt{3}}\,\sigma_2\wedge\sigma_3 - \frac{3^{3/4}\,y^2}{4\times 2^{1/6}}\,\mathrm{d}u\wedge\sigma_2 + \frac{y}{\sqrt{3}}\,\mathrm{d}y\wedge\sigma_1\right)\,, \quad (49)$$

which allows one easily to extract the corresponding Penrose limits of the NS-NS 2-form B_2 and the R-R 2-form potential C_2, namely

$$\tilde{B}_2 = \frac{y^2}{\sqrt{3}}\,\sigma_2\wedge\sigma_3 - \frac{3^{3/4}\,y^2}{4\times 2^{1/6}}\,\mathrm{d}u\wedge\sigma_2 + \frac{y}{\sqrt{3}}\,\mathrm{d}y\wedge\sigma_1\,. \quad (50)$$

$$\tilde{C}_2 = \frac{y^2}{\sqrt{3}}\,\sigma_1\wedge\sigma_3 - \frac{3^{3/4}\,y^2}{4\times 2^{1/6}}\,\mathrm{d}u\wedge\sigma_1 - \frac{y}{\sqrt{3}}\,\mathrm{d}y\wedge\sigma_2\,, \quad (51)$$

For simplicity we will refer to the components of the B-field as $\tilde{B}_{\sigma_2\sigma_3}$, $\tilde{B}_{u\sigma_2}$ and $\tilde{B}_{y\sigma_1}$, with values given in (50). For the limit of the F_3 flux in the sigma co-frame one finds

$$\tilde{F}_3 = \tilde{F}_3^{(1)}\,\mathrm{d}u\wedge\mathrm{d}y\wedge\sigma_1 + \tilde{F}_3^{(2)}\,\mathrm{d}u\wedge\sigma_2\wedge\sigma_3\,, \quad (52)$$

where

$$\tilde{F}_3^{(1)} = \frac{3^{3/4}\,y}{2\times 2^{1/6}},\quad \tilde{F}_3^{(2)} = \frac{3^{3/4}\,y^2}{2\times 2^{1/6}}\,. \quad (53)$$

[4]Here we have introduced a tilde for the pp-wave metric. From now on it will be convenient to use tilde for the pp-wave quantities, while we will use widehat for the NAT-dual quantities.

Finally, the Penrose limit of the 7-form flux (43) gives

$$\begin{aligned}\tilde{F}_7 &= \tilde{F}_7^{(1)}\,\mathrm{d}u\wedge\mathrm{d}\phi_1\wedge\mathrm{d}\phi_2\wedge\mathrm{d}\phi_3\wedge\mathrm{d}r\wedge\mathrm{d}y\wedge\sigma_1\\ &\quad+\tilde{F}_7^{(2)}\,\mathrm{d}u\wedge\mathrm{d}\phi_1\wedge\mathrm{d}\phi_2\wedge\mathrm{d}\phi_3\wedge\mathrm{d}r\wedge\sigma_2\wedge\sigma_3\,,\end{aligned}\tag{54}$$

with the following components:

$$\tilde{F}_7^{(1)}=-\frac{3^{3/4}\,r^3\,y\sin^2\phi_1}{2\times 2^{1/6}},\quad \tilde{F}_7^{(2)}=-\frac{3^{3/4}\,r^3\,y^2\sin^2\phi_1}{2\times 2^{1/6}}\,.\tag{55}$$

In the following Section we apply the NATD procedure from [5] along the $SU(2)$ isometry of the pp-wave metric (46).

5 The NATD for the NS-NS sector

The solution (46) retains the $SU(2)$ invariance of the original PW metric (28), so we can apply the NATD procedure given in [5], which we adapted for the left invariant $SU(2)$ one-forms from eq. (32). The result is the following T-dual metric:

$$\begin{aligned}\mathrm{d}\hat{s}^2 &= -2\,\mathrm{d}u\,\mathrm{d}v+\widehat{G}_{uu}\,\mathrm{d}u^2+2\,\widehat{G}_{uy}\,\mathrm{d}u\,\mathrm{d}y+\widehat{G}_{yy}\,\mathrm{d}y^2\\ &\quad+2\sum_{a=1}^{3}\widehat{G}_{ux_a}\,\mathrm{d}u\,\mathrm{d}x_a+2\sum_{a=1}^{3}\widehat{G}_{yx_a}\,\mathrm{d}y\,\mathrm{d}x_a+\sum_{a=1}^{3}\widehat{G}_{x_ax_a}\,\mathrm{d}x_a^2\\ &\qquad+\sum_{a\neq b=1}^{3}\widehat{G}_{x_ax_b}\,\mathrm{d}x_a\,\mathrm{d}x_b+\mathrm{d}r^2+r^2\,\mathrm{d}\Omega_3^2\,,\end{aligned}\tag{56}$$

with components given by

$$\begin{aligned}
\widehat{G}_{uu} &= -\frac{\sqrt{3}\,r^2}{4\times 2^{1/3}}-\frac{y^4\left(3\sqrt{3}\,x_1^2+6\,x_1\,y^2+\sqrt{3}\left(4\,x_3^2+y^4\right)\right)}{4\times 2^{1/3}\,M},\\
\widehat{G}_{uy} &= \widehat{G}_{yu}=-\frac{x_2\,y^3\left(3\,x_1+\sqrt{3}\,y^2\right)}{2^{1/6}\,3^{3/4}M}\,,\quad \widehat{G}_{yy}=\frac{4}{3}-\frac{4\left(x_2^2+x_3^2\right)y^2}{3\,M}\,,\\
\widehat{G}_{ux_1} &= \widehat{G}_{x_1u}=-\frac{y^2\left(6x_1\,x_2+\left(\sqrt{3}\,x_2-3\,x_3\right)y^2\right)}{2\times 2^{1/6}\,3^{1/4}\,M}\,,\\
\widehat{G}_{ux_2} &= \widehat{G}_{x_2u}=-\frac{3^{3/4}\,y^2\left(4\,x_2^2+y^4\right)}{4\times 2^{1/6}\,M}\,,\\
\widehat{G}_{ux_3} &= \widehat{G}_{x_3u}=-\frac{y^2\left(12\,x_2\,x_3+6\,x_1\,y^2+\sqrt{3}\,y^4\right)}{4\times 2^{1/6}\,3^{1/4}\,M}\,,\\
\widehat{G}_{yx_1} &= \widehat{G}_{x_1y}=\frac{4\,y\left(3\sqrt{3}\,x_1^2+3\,x_1\,y^2+\sqrt{3}\,y^4\right)}{9\,M}\,,\\
\widehat{G}_{yx_2} &= \widehat{G}_{x_2y}=\frac{4\sqrt{3}\,x_1\,x_2\,y+2\left(x_2+\sqrt{3}\,x_3\right)y^3}{3\,M}\,,
\end{aligned}$$

$$
\begin{aligned}
&\widehat{G}_{yx_3} = \widehat{G}_{x_3y} = \frac{2\,y\,\left(6\,x_1\,x_3 + \left(\sqrt{3}\,x_3 - 3\,x_2\right)\,y^2\right)}{3\,\sqrt{3}\,M}\,,\\
&\widehat{G}_{x_1x_1} = \frac{4(3x_1^2+\sqrt{3}x_1y^2+y^4)}{3M}\,, \quad \widehat{G}_{x_2x_2} = \frac{4x_2^2+y^4}{M}\,, \quad \widehat{G}_{x_3x_3} = \frac{4x_3^2+y^4}{M}\,,\\
&\widehat{G}_{x_1x_2} = \widehat{G}_{x_2x_1} = \frac{2x_2(6x_1+\sqrt{3}y^2)}{3M}\,, \quad \widehat{G}_{x_1x_3} = \widehat{G}_{x_3x_1} = \frac{2x_3(6x_1+\sqrt{3}y^2)}{3M}\,,\\
&\widehat{G}_{x_2x_3} = \widehat{G}_{x_3x_2} = \frac{4\,x_2\,x_3}{M}\,,
\end{aligned}
$$

and

$$
M = \frac{4}{3}\,y^2\,\left(3\,\left(x_1^2+x_2^2+x_3^2\right) + \sqrt{3}\,x_1\,y^2 + y^4\right)\,. \tag{57}
$$

The dual B-field has the following form:

$$
\widehat{B}_2 = \widehat{B}_{uy}\,\mathrm{d}u\wedge\mathrm{d}y + \sum_{a=1}^{3}\widehat{B}_{ux_a}\,\mathrm{d}u\wedge\mathrm{d}x_a + \frac{1}{2}\sum_{a,b=1}^{3}\widehat{B}_{x_ax_b}\,\mathrm{d}x_a\wedge\mathrm{d}x_b\,, \tag{58}
$$

where

$$
\begin{aligned}
&\widehat{B}_{uy} = -\widehat{B}_{yu} = \frac{x_3\,y^3\,\left(\sqrt{3}\,x_1 - y^2\right)}{2^{1/6}\,3^{3/4}\,M}\,,\\
&\widehat{B}_{ux_1} = -\widehat{B}_{x_1u} = \frac{y^2\,\left(6\,x_1\,x_3 + \left(3\,x_2 + \sqrt{3}\,x_3\right)\,y^2\right)}{2\times 2^{1/6}\,3^{3/4}\,M}\,,\\
&\widehat{B}_{ux_2} = -\widehat{B}_{x_2u} = -\frac{y^2\,\left(-12\,x_2\,x_3 + 6\,x_1\,y^2 + \sqrt{3}\,y^4\right)}{4\times 2^{1/6}\,3^{3/4}\,M}\,,\\
&\widehat{B}_{ux_3} = -\widehat{B}_{x_3u} = \frac{3^{1/4}\,y^2\,\left(4\,x_3^2 + y^4\right)}{4\times 2^{1/6}\,M}\,,\\
&\widehat{B}_{x_1x_2} = -\widehat{B}_{x_2x_1} = -\frac{2\,x_3\,y^2}{M}\,, \quad \widehat{B}_{x_1x_3} = -\widehat{B}_{x_3x_1} = \frac{2\,x_2\,y^2}{M},\\
&\widehat{B}_{x_2x_3} = -\widehat{B}_{x_3x_2} = -\frac{y^2\,\left(6\,x_1 + \sqrt{3}\,y^2\right)}{3\,M}\,.
\end{aligned}
$$

As expected the expressions for the dual metric and B-field are fairly much more complicated than their initial counterparts before the NATD procedures.

6 The NATD for the R-R sector

One can also apply the Fourier-Mukai on the pp-wave R-R forms, which leads to the following non-trivial NAT-Dual R-R fluxes: $\widehat{F}_2$, $\widehat{F}_4$, $\widehat{F}_6$, and $\widehat{F}_8$, while $\widehat{F}_{10}=0$. The result is listed below. The 2-form flux is

$$
\widehat{F}_2 = \widehat{F}_2^{(1)}\,\mathrm{d}u\wedge\mathrm{d}y - \tilde{F}_3^{(2)}\,\mathrm{d}u\wedge\mathrm{d}x_1\,, \tag{59}
$$

where the coefficients with the tilde are defined in Section 3. The wide-hatted coefficient is

$$\widehat{F}_2^{(1)} = \Big(2\,x_1 - \tilde{B}_{\sigma_2\sigma_3}\Big)\;\tilde{F}_3^{(1)} - \tilde{B}_{y\sigma_1}\,\tilde{F}_3^{(2)} + \tilde{F}_5^{(2)}\,. \tag{60}$$

The 4-form flux is

$$\begin{aligned}\widehat{F}_4 &= \widehat{F}_4^{(1)}\,\mathrm{d}u\wedge\mathrm{d}x_1\wedge\mathrm{d}x_2\wedge\mathrm{d}x_3 + \widehat{F}_4^{(2)}\,\mathrm{d}u\wedge\mathrm{d}y\wedge\mathrm{d}x_1\wedge\mathrm{d}x_2 \\ &\quad + \widehat{F}_4^{(3)}\,\mathrm{d}u\wedge\mathrm{d}y\wedge\mathrm{d}x_1\wedge\mathrm{d}x_3 + \widehat{F}_4^{(4)}\,\mathrm{d}u\wedge\mathrm{d}y\wedge\mathrm{d}x_2\wedge\mathrm{d}x_3\,.\end{aligned} \tag{61}$$

where

$$\begin{aligned}&\widehat{F}_4^{(1)} = -\widehat{B}_{x_2x_3}\,\tilde{F}_3^{(2)}\,,\quad \widehat{F}_4^{(2)} = \widehat{B}_{x_1x_2}\,\widehat{F}_2^{(1)}\,,\\ &\widehat{F}_4^{(3)} = \widehat{B}_{x_1x_3}\,\widehat{F}_2^{(1)}\,,\quad \widehat{F}_4^{(4)} = -\tilde{F}_3^{(1)} + \widehat{B}_{x_2x_3}\,\widehat{F}_2^{(1)}\,.\end{aligned}$$

The 6-form field strength is

$$\begin{aligned}\widehat{F}_6 &= \widehat{F}_6^{(1)}\,\mathrm{d}u\wedge\mathrm{d}\phi_1\wedge\mathrm{d}\phi_2\wedge\mathrm{d}\phi_3\wedge\mathrm{d}r\wedge\mathrm{d}x_1\\ &\quad - 2\,x_2\,\tilde{F}_5^{(1)}\,\mathrm{d}u\wedge\mathrm{d}\phi_1\wedge\mathrm{d}\phi_2\wedge\mathrm{d}\phi_3\wedge\mathrm{d}r\wedge\mathrm{d}x_2\\ &\quad - 2\,x_3\,\tilde{F}_5^{(1)}\,\mathrm{d}u\wedge\mathrm{d}\phi_1\wedge\mathrm{d}\phi_2\wedge\mathrm{d}\phi_3\wedge\mathrm{d}r\wedge\mathrm{d}x_3\\ &\quad + \widehat{F}_6^{(2)}\,\mathrm{d}u\wedge\mathrm{d}\phi_1\wedge\mathrm{d}\phi_2\wedge\mathrm{d}\phi_3\wedge\mathrm{d}r\wedge\mathrm{d}y\,,\end{aligned} \tag{62}$$

where

$$\begin{aligned}&\widehat{F}_6^{(1)} = -\Big(2\,x_1 - \tilde{B}_{\sigma_2\sigma_3}\Big)\;\tilde{F}_5^{(1)} - \tilde{F}_7^{(2)}\,,\\ &\widehat{F}_6^{(2)} = \Big(2\,x_1 - \tilde{B}_{\sigma_2\sigma_3}\Big)\;\Big(\tilde{F}_7^{(1)} - \tilde{B}_{y\sigma_1}\,\tilde{F}_5^{(1)}\Big) - \tilde{B}_{y\sigma_1}\,\tilde{F}_7^{(2)}\,.\end{aligned}$$

Finally, the 8-form flux is given by

$$\begin{aligned}\hat{F}_8 &= \widehat{F}_8^{(1)}\,\mathrm{d}u\wedge\mathrm{d}\phi_1\wedge\mathrm{d}\phi_2\wedge\mathrm{d}\phi_3\wedge\mathrm{d}r\wedge\mathrm{d}y\wedge\mathrm{d}x_1\wedge\mathrm{d}x_2\\ &\quad + \widehat{F}_8^{(2)}\,\mathrm{d}u\wedge\mathrm{d}\phi_1\wedge\mathrm{d}\phi_2\wedge\mathrm{d}\phi_3\wedge\mathrm{d}r\wedge\mathrm{d}y\wedge\mathrm{d}x_1\wedge\mathrm{d}x_3\\ &\quad + \widehat{F}_8^{(3)}\,\mathrm{d}u\wedge\mathrm{d}\phi_1\wedge\mathrm{d}\phi_2\wedge\mathrm{d}\phi_3\wedge\mathrm{d}r\wedge\mathrm{d}y\wedge\mathrm{d}x_2\wedge\mathrm{d}x_3\\ &\quad + \widehat{F}_8^{(4)}\,\mathrm{d}u\wedge\mathrm{d}\phi_1\wedge\mathrm{d}\phi_2\wedge\mathrm{d}\phi_3\wedge\mathrm{d}r\wedge\mathrm{d}x_1\wedge\mathrm{d}x_2\wedge\mathrm{d}x_3\,.\end{aligned} \tag{63}$$

with the following components:

$$\begin{aligned}&\widehat{F}_8^{(1)} = \widehat{B}_{x_1x_2}\,\widehat{F}_6^{(2)}\,,\quad \widehat{F}_8^{(2)} = \widehat{B}_{x_1x_3}\,\widehat{F}_6^{(2)}\,,\\ &\widehat{F}_8^{(3)} = \big(\tilde{B}_{y\sigma_1}\,\tilde{F}_5^{(1)} - \tilde{F}_7^{(1)}\big) + \widehat{B}_{x_2x_3}\,\widehat{F}_6^{(2)}\,,\\ &\widehat{F}_8^{(4)} = \big(2x_2\hat{B}_{x_1x_3} - 2x_3\hat{B}_{x_1x_2} - \big(2x_1 - \tilde{B}_{\sigma_2\sigma_3}\big)\hat{B}_{x_2x_3} + 1\big)\tilde{F}_5^{(1)} - \widehat{B}_{x_2x_3}\tilde{F}_7^{(2)}\,.\end{aligned}$$

Finally, the last type IIA dual field strength F_{10} vanishes.

7 Conclusion

Plane waves or pp-wave spacetimes are solutions that model radiation moving at the speed of light and may consist of electromagnetic or gravitational radiation. The interest of considering pp-wave geometries is that they may become invaluable tool for studying the far Universe, especially after the recent discovery of gravitational waves [10, 11]. An important feature of string theory on such backgrounds is that it is exactly solvable, which can provide insights in many aspects of the theory in more complicated geometries especially if they are dual to non-conformal gauge theories.

Furthermore string theory exibits numerous dualities between seemingly different theories, which give us powerful new ways to look at different physical phenomena. One such duality is T-duality, which relates the weak coupling sectors of two string theories. The key point here is that these theories live in spacetimes with reciprocal radii of their compact dimensions.

Recently, the authors of [5] derived the T-duality transformation rules for backgrounds possessing non-abelian $SU(2)$ G-structure, which is the setup we adopt in this work. Here we consider non-abelian T-duality from the Penrose limit of the Pilch-Warner supergravity solution. This limit also contains non-trivial NS and R-R fluxes. It is crucial for us that the new Penrose limit retains the $SU(2)$ G-structure of the original solution. This fact facilitates the calculation of its non-abelian T-dual counterpart.

Following [5] we obtain the ten-dimensional non-abelian T-dual of a certain pp-wave limit of the Pilch-Warner metric together with the dual NS Kalb-Ramond B-field. The result for the dual metric and B-field is relatively complicated, because the application of the NATD procedure mixes the coordinates in a non-trivial way.

We also obtain the type IIA dual field strengths of the R-R fluxes via application of the Fourier-Mukai transform. The resulting expressions for the dual fluxes are also complicated, which is an expected property of the NATD procedure.

Obviously the NATD procedure generates complicated geometries even if we consider the simpler case of pp-waves. One interesting question that arise here is do these procedures (NATD and PL) commute. Although we expect that interchanging the order of the procedures will lead to different solutions, this assumption needs more careful analysis, which we hope to deliver soon.

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