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New Approach in the Study of Processes in Solar-Terrestrial Physics

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Abstract. A new mathematical approach for study of physical processes into the system Sun – Earth is introduced in the present work. The introduction and application of structures as morphism, group, category, monad, functor, natural transformation and others is appropriate when calculating parameters of space weather as cosmic rays, solar wind, interplanetary magnetic fields, Earth's magnetosphere, geomagnetic storms and others. Such problems are principally considered in the proposed work.

1 Introduction

The actual problems of solar-terrestrial physics, in particular of space weather are related to the prediction of the space environment state and are solved by means of different analyses and models. In this work is introduced a new mathematical approach [1] to study of physical processes into the system Sun - Earth. For example, by calculation of the ionization $q(h)$ profiles in the ionosphere and atmosphere under the influence of galactic cosmic rays (GCR) and anomalous CR (ACR) a model is used that applies the principle of isomorphism [1,2].

When calculating the parameters of space weather such as GCR, ACR, solar wind, interplanetary magnetic fields, Earth's magnetosphere, geomagnetic storms and others, it is appropriate the introduction and application of algebraic structures: morphism, category, functor, natural transformation and others [1,2].

Such an approach would take account of the general laws of physical processes into the system Sun-Earth [3] and help in their testing and calculation. This approach would be useful for such complex systems and processes as these in the solar-terrestrial physics and space weather [4,5]. Here some methods for algebraic data structures could be introduced. These methods give the possibility for axiomatization of the physical data reality and the application of algebraic methods for their processing. Here is given the base for the transformation from the algebraic theory of category and morphisms [1,2] to the physical structure of concepts and data.

2 Morphisms and Categories and Solar-Terrestrial Physics

Let $\square$ be binary operation in a set X and $\square'$ is another such operation in a set X'. A morphism f: $(X, \square) \to (X', \square')$ is a function from X to X' which transfers the operation $\square$ in X to the operation $\square'$ in X' so that

$$f(x \square y) = (fx \square' fy)$$

for all $x, y \in X$ [1,2].

The axioms and definitions from categorical algebra present the base of the new paradigm which describes the connection between theory of category and morphisms and solar-terrestrial physics. The definition of category shows its strong relation to the notion of morphism. On this way the morphisms are interpreted as images from one physical object to another so that the properties of the first object influence on the properties of the second and from the data of the first object some information can be obtained for the second object [6].

3 Interval Ionization from CR as Category with Two Objects

The isomorphism $q(x \cup y) = qx + \ qy$ presents interval ionization from cosmic rays (CR) in the planetary ionosphere-atmosphere. Here x, y are energy intervals in the CR ionization losses, i.e. the formula of Bohr-Bethe-Bloch, and q is function which is bijection [1]. The mathematical expression for q is the following [3-5]:

$$q_i(h) = \frac{1}{Q} \int\limits_{E_i}^{\infty} \int\limits_{\phi=0}^{2\pi} \int\limits_{\theta=0}^{\pi/2+\Delta\theta} D_i(E,h) \left[\frac{dE}{dh}(h)\right]_i \sin\theta \, d\theta \, d\phi \, dE \qquad (1)$$

Here $Q = 35$ eV is the energy required for obtaining an electron-ion pair, E_i are geomagnetic cut-offs, φ – the azimuth angle of penetration of CRs, θ – the zenith angle, $D_i(E)$ – the differential spectrum of CR from type i, $\left[\frac{dE}{dh}(h)\right]_i$ – ionization losses at height h, E is the kinetic energy of CR.

Different categories $C(X, Y)$ are obtained for different GCR particles (proton p, alpha α, light L, medium M, heavy H, very heavy VH and super heavy SH groups of nuclei [3-5]) for diverse planets and satellites in the Solar system (Earth, Mars, Venus, Jupiter, Saturn, Titan, Uranus, Neptune etc.), where appear varied models of neutral atmosphere, planetary oblateness and magnetic cut-offs (for different latitudes and longitudes) in their environments. For the case of unity of three intervals [7] (with charge decrease interval for charge $Z > 1$) the morphism is obtained analogically.

It becomes clear that the mathematical expression (1) satisfies the category definition, given above, for the case of two objects (X, Y). The object X is the set of characteristic energy intervals in the ionization losses function. The object Y is the set of the interval ionizations which are caused by the cosmic rays with energy intervals in object X [1,2]. The reversed morphism to the morphism $\mathrm{Hom}(X, Y)$ is also fulfilled, namely $f^{-1}(x + y) = f^{-1}(x)\;(f^{-1}(y) = I_1(I_2$. The corresponding unity of both intervals $I_1(I_2$ is obtained [1-4]. This analysis can be extended in the case of one, three and four objects when we introduce the notion of monoid, the interval integral spectrum and the interval ozone production on the base of interval ionization in the Earth's environment [1,3,4]. Our results for formation of categories and for ionization profiles due to GCR and ACR are shown in Figures 1-5.

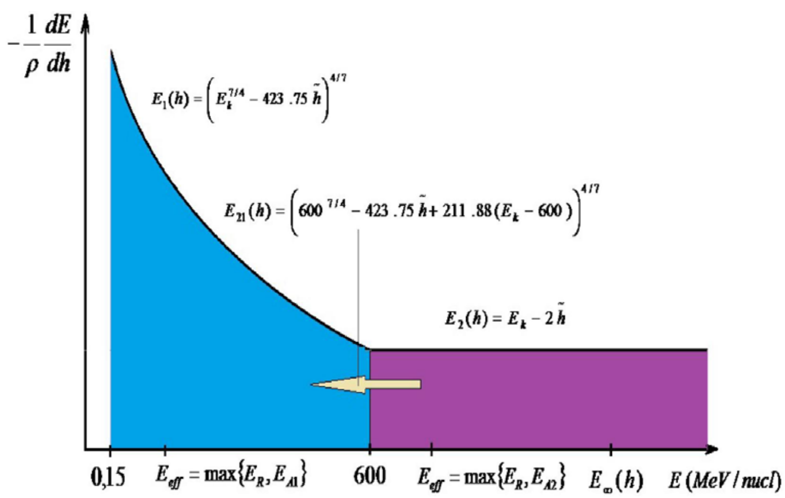

Figure 1: Formation of a category with one object for two-interval approximation of the ionization losses function for GCR protons ($Z = 1$). This picture from [7] is used here for explanation of our new mathematical approach.

4 Discussion and Conclusion

From the completed analysis and from application examples it can be concluded that the theory of categories and morphisms is applicable for the processes of solar-terrestrial physics and space weather. The mor-

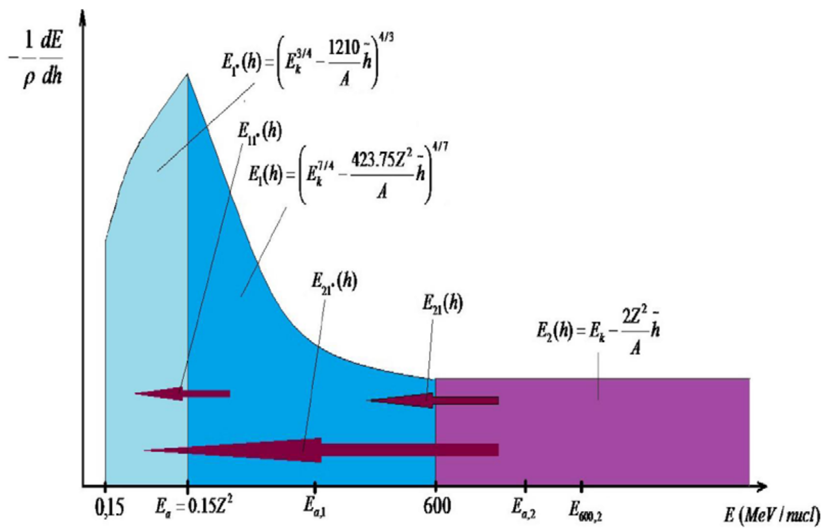

Figure 2: Formation of a category with one object for three-interval approximation of the ionization losses function for heavier GCR particles with $Z > 1$ (α, L, M, H, VH, SH groups of nuclei). This picture from [7] is used here for explanation of our new mathematical approach.

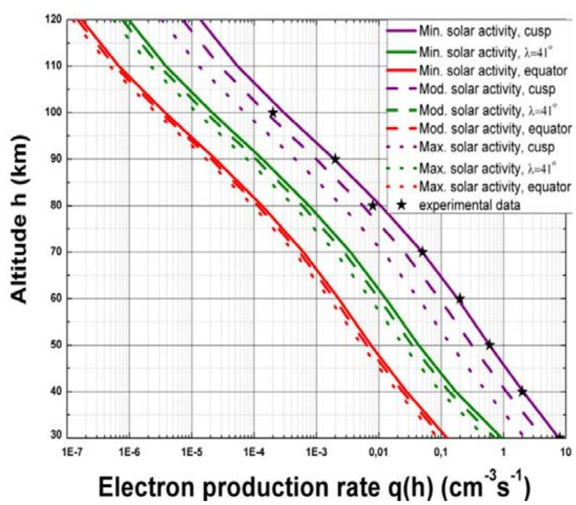

Figure 3: Electron production rate $q(h)$ profiles due to GCR (p, α, L, M, H, VH, SH groups of nuclei) for cusp, middle latitudes and equator at minimal, moderate and maximal solar activity. Experimental data are shown with (*).

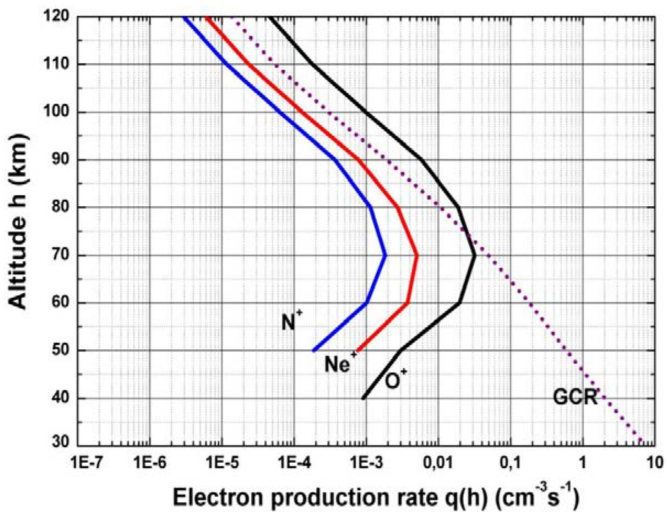

Figure 4: Electron production rate $q(h)$ profiles calculated with program CORIMIA for main ACR constituents: Nitrogen (N), Oxygen (O) and Neon (Ne) (Leske et al. 2011). GCR ionization $q(h)$ is at minimal solar activity.

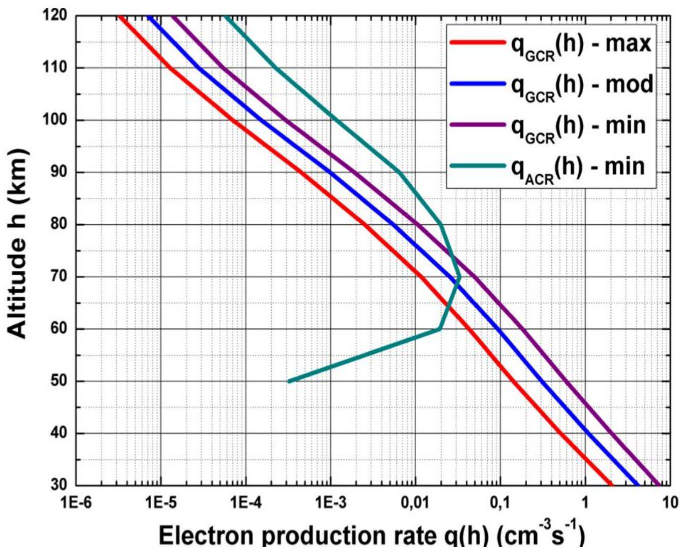

Figure 5: Electron production rates $q(h)$ in cusp computed with the program CORIMIA. This is the resultant ACR profile (N + O + Ne, from Figure 4) compared with the GCR contribution. GCR ionization is shown for conditions of minimal, moderate and maximal solar activity (Figure 3).

phisms from the conditions for category existence are fulfilled, as also the conditions themselves are satisfied – i.e. the corresponding categories are generated. It can be supposed that different generalizations and extensions of this analysis are fully possible and applicable, because the definitions for morphism, category, natural transformation, group etc. are mutually related and these notions are mutually generated.

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